NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

DEVELOPMENT OF REGIONAL EXTREME MODEL ATMOSPHERES
BY REGRESSION METHODS

by

Donald Anthony Quinn

Thesis Advisor: F. L. Martin

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Development of Regional Extreme Model Atmospheres by Regression Methods

by

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Lieutenant, United States Navy
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ABSTRACT

A group of stations in the North American Arctic region have been analyzed for statistical determination of temperatures at mandatory-pressure levels. For each station the temperature at a key level, peculiar to that station, has been forced in at the first step and retained at each subsequent step, in the development of stepwise regression equations which predict temperatures at the mandatory levels. In general, eight-step predictions, in terms of inter-level thicknesses, were found to give optimum specification of the desired temperatures. The best estimate of the regional atmosphere, which is conditionally dependent upon the existence of an extreme 1% probability of the forcing-level temperature, is obtained with a high degree of confidence.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES (continued)

LIST OF FIGURES

TABLE OF SYMBOLS AND ABBREVIATIONS

AFCRL	Air Force Cambrdige Research Laboratory
C.E.V.	Cumulative explained variance
COESA	U.S. Committee on Extension to the Standard Atmosphere
^{o}C	Degrees Celsius
F_r	F-ratio upon entry at step r
i	Sample-element index
J	Forcing-level index
k	Data-level index
mb	Millibar
MID-STD	Military Standard
n	10% sample size
N	Full-sample size
NWT	Northwest Territory
P_J	Forcing-level pressure
P_k	Data-level pressure
P_M	Mandatory-pressure level
r	The number of predictors selected
R	Multiple correlation coefficient
S.E.	Standard error of estimate
σ_E	Standard error of estimate
σ_J	Standard deviation at the forcing level
$T_J(P_J)$	Forcing-level temperature at P_J
$\overline{T}_J(\ )$	Mean forcing-level temperature
$T_M(\ )$	Mandatory-level temperature
U.E.V.	Unexplained variance

TABLE OF SYMBOLS AND ABBREVIATIONS (continued)

X_M	Structure function of the regression equations
z_k	Height of pressure level in geopotential meters at P_k
Z_k	Inter-level thickness between P_k and P_{k+1}

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I. INTRODUCTION

Although many factors must be considered in determining the thermal environment of airborne ordnance, the most dominant are the flight conditions and the atmospheric temperature profile. While aircraft and missile performance capabilities are well defined, there is a great deal of variability in the atmospheric temperature profile which may be traversed by the vehicle. Therefore many model atmospheres have been developed for meteorological purposes, but none is completely satisfactory for use in aerothermodynamic heating calculations.

Supplemental standard atmospheres have been developed by COESA(1966) to provide information on spatial and temporal variability across selected latitude bands and by seasons. However the supplement standards are intended to be means with respect to the latitude bands and seasons, and are not representative of the extremes which must be considered for aerodynamic design purposes. Richard and Snelling (1971) proposed the adoption of a revised MIL-STD-210 hot and cold model atmospheres which represent global extreme values at successive 2 km elevations. These extremes are deviations from the station mean at the elevation involved, which are exceeded at the 1%, 5% and 10% probability levels, and were derived through analysis of extreme data from global meteorological temperature maps analyzed at constant pressure levels. More recently a second draft, MIL-STD-210B, has been published (1972). Both of these extreme sets of profiles pose serious design problems when changes of elevation are involved.

The most important limitation of the two recently proposed extreme-atmosphere sets is that they cannot be realistically utilized in conjunction with flight profiles and trajectories which involve changes in altitude. The temperature extremes at differing altitudes are in most cases representative of different geographical locations rather than of conditions occurring in real time over a real location. Polar and tropical standard and extreme atmospheres have been developed for such applications but are still subject to the deficiencies noted above.

TABLE 1. Proposed locations of the 1% world-wide extreme temperatures at indicated levels (AFCRL, 1970).

Level	Pressure altitude (ft)	1% Cold extreme location	1% Warm extreme location
SFC	0	Oymyakon, USSR	Insalah, Algeria
850 mb	4780	Oymyakon, USSR	Insalah, Algeria
700 mb	9882	Hall Beach, NWT *	Babylon, Iraq
500 mb	18287	Resolute, NWT *	New Delhi, India
300 mb	30066	Thule, Greenland*	New Delhi, India
200 mb	38661	Thule, Greenland*	Alert, NWT *
150 mb	44646	Karachi, Pakistan	Alert, NWT *
100 mb	53084	Singapore	Thule, Greenland*

The object of this study was to develop the most probable vertical profile, conditional with the existence of a 1% extreme at a given level, for each station set forth in Table 1 (but here limited to those marked by an asterisk). Each station of Table 1 has been associated with a listed 1% extreme for the indicated month according to a preliminary draft of the MIL-STD-210B proposed atmosphere, which is graphically illustrated in Fig. 1. The asterisk-marked stations all occur in the Canadian Arctic and are either extremely cold at the

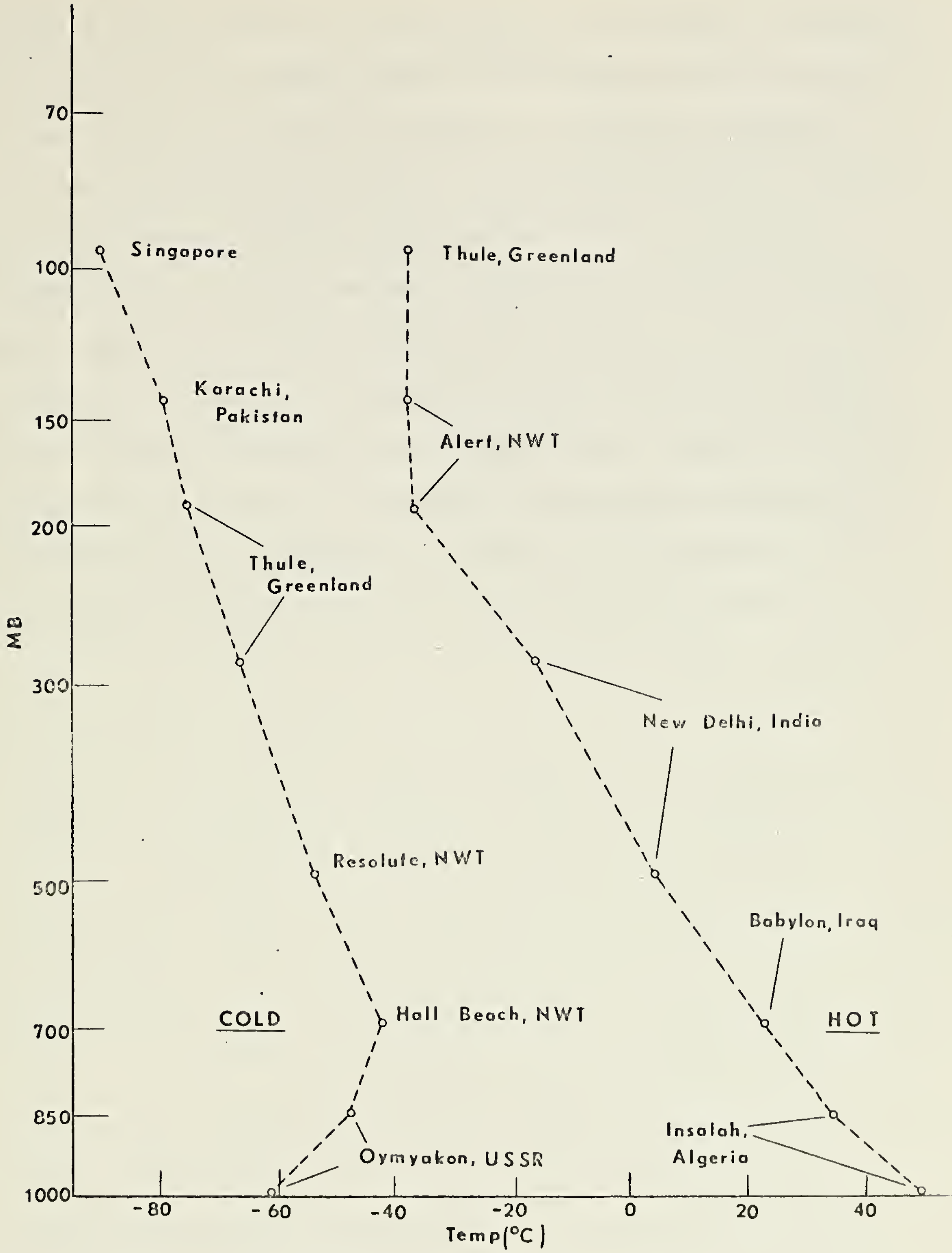

Fig. 1. Proposed MIL-STD-210B atmospheres

indicated level in winter or extremely warm in the summer stratosphere. This subset of the stations in Table 1 was isolated from the full set because the data were readily available in conveniently summarized form from checked rawinsonde data. The financial assistance and kind cooperation of the Commander, Naval Weather Service is gratefully acknowledged in acquiring the data on magnetic tape from the National Environmental Data Service, Ashville, North Carolina. For each station and month of interest, four years (1967-70) of atmospheric soundings were processed to determine the temperature means, standard deviations and inter-level correlations required for application of the multiple regression analysis procedure to be used in the determination of the most probable temperature profile at the 1% probability level.

II. DATA PROCESSING

For each of the stations marked with an asterisk in Table 1, four years of checked rawinsonde data, for the period 1967 through 1970, was provided by the Naval Weather Service Environmental Detachment, Ashville. Those stations listed under the cold extreme of Table 1 were subject only to January data samples, and those under the warm extreme, only to July samples. The individual sounding samples were arranged in the sequential format indicated in Table 2 and recorded on magnetic tape. Height and temperature data for the indicated pressure levels were arranged with six sets in each of the first three rows and five sets in the fourth row. Note that the data is arranged at 50 mb increments from 1000 to 200 mb inclusively and at somewhat smaller increments thereafter.

TABLE 2. Arrangement of sounding temperature and geopotential height data by pressure levels k=1,...,23 in each sounding.

Index k =	1	2	3	4	5	6
level (mb)	1000	950	900	850	800	750
Index k =	7	8	9	10	11	12
level (mb)	700	650	600	550	500	450
Index k =	13	14	15	16	17	18
level (mb)	400	350	300	250	200	175
Index k =	19	20	21	22	23	
level (mb)	150	125	100	80	70	

With the data in the format of Table 2, it is convenient to determine the mean temperature $\overline{T}(P_J)$ and standard deviation σ_J for the

indicated station levels specified by Table 1. The temperature $T(P_J)$ is to be used in a multiple regression analysis to determine the most probable vertical temperature profile at all mandatory-sounding levels associated with that "forcing-level temperature" $T(P_J)$. A subset of such station data is further analyzed for determination of the above statistical parameters for those soundings having 10% and 1% extreme temperatures at the forcing level. The $T_J(.10)$ and $T_J(.01)$ extreme temperatures can then be forced into the derived full-sample regression procedure to obtain the corresponding extreme profiles.

TABLE 3. Nominal temperature extremes at indicated stations and pressure levels at the 10% and 1% values of a normal distribution.

Station	Forcing Level T_J	$T_J(.10)$	$T_J(.01)$	Full Sample Size	Sample Size 10% Extreme
January Cases					
Hall Beach	T(700)	-35.6	-42.3	243	26
Resolute	T(500)	-47.1	-51.3	237	25
Thule	T(300)	-62.0	-67.3	213	13
Thule	T(200)	-63.5	-70.6	213	18
July Cases					
Thule	T(100)	-41.3	-39.8	236	23
Alert	T(150)	-39.7	-37.9	244	22
Alert	T(200)	-38.6	-35.7	244	18

Table 3 lists the forcing level pressures and sample sizes with the 10% and 1% limiting temperatures for each station under considera-tion. Estimates of the temperatures $T(P_J)$, at the probability extremes

of 10% and 1% have been computed assuming a Gaussian distribution at each J-level. These estimates, $T_J(.10)$ and $T_J(.01)$ of Table 3, are derived using the computed full sample-means $\overline{T}(P_J)$ and standard deviations σ_J of the 1967-70 samples together with the well-known relationships

$$\begin{aligned} T_J(.10) &= \overline{T}_J \pm 1.2817\sigma_J \\ T_J(.01) &= \overline{T}_J \pm 2.3267\sigma_J \end{aligned} \tag{1}$$

where the plus sign is to be used for a warm extreme and the minus sign for a cold extreme.

Analysis of the data revealed an insufficient number of soundings occurring at the 1% extreme of $T(P_J)$ for a test of the statistical significance of these results. It was decided therefore, to give primary consideration to the 10% extreme $T(P_J)$ soundings and to deduce the properties of the 1% subset as a limiting case of the 10% sample. Thus the same multiple regression treatment used to predict the $T(P_M)$ mandatory-level temperatures for the full sample has been performed upon the "10% subset" containing $T(P_J)$ extremes.

III. THE REGRESSION METHOD

A. THE STEPWISE REGRESSION PROCEDURE

Many problems in research require the extensive analysis of large amounts of data, and the process for handling it should be made as automatic and rapid as possible. To this end, the Biomedical Computer Programs (Dixon, 1966) were developed. One of these programs, BIMED 02R, provides stepwise regression analysis capability and will be utilized here.

BIMED 02R computes, in a stepwise manner, a sequence of multiple linear regression equations. That single variable is added to the equation at each step which produces the greatest reduction in the previous unexplained variance. This variable is also the one which has the highest partial correlation with the dependent variable at the particular step in the analysis of the variance. Equivalently it is the variable which would give the highest F-statistic upon entry at that step. The F-statistic upon entry F_r, is expressed at step r as (Dixon 1966)

$$F_r(1,n-r-1) = \frac{\%\ (C.E.V.,r) - \%\ (C.E.V.,r-1)}{\%\ (U.E.V.,r)}$$

where

% (C.E.V.,r) is the percent cumulative explained variance at step r,

% (C.E.V.,r-1) is the percent cumulative explained variance at step r-1,

% (U.E.V.,r) is the percent unexplained variance remaining at step r.

This study utilized a statistical model expressible in the form

$$T_M = C_0 + C_1 T(P_J) + C_2 X_M \tag{2}$$

Here T_M in (2) is the temperature predictand at each of the eight mandatory levels chosen alternately as

$$T_M = T_1,\ T_2,\ T_3,\ T_4,\ T_5,\ T_6,\ T_7,\ T_8, \text{ and } T_9$$

but with $T_M \neq T(P_J)$, since the forcing-level temperature is used as the forced predictor for each of the eight mandatory level T_M specifications for the station under consideration. Thus for each of the stations in Table 3, eight of the following nine dependent variables were specified for use in the multiple regression procedure:

T_1, the temperature at 1000 mb, T(1000)

T_2, the temperature at 850 mb, T(850)

T_3, the temperature at 700 mb, T(700)

T_4, the temperature at 500 mb, T(500)

T_5, the temperature at 300 mb, T(300)

T_6, the temperature at 200 mb, T(200)

T_7, the temperature at 150 mb, T(150)

T_8, the temperature at 100 mb, T(100)

T_9, the temperature at 70 mb, T(70).

$T(P_J)$ is forced into the specification of each of the eight possible predictands in the manner symbolized by Eq. (2). X_M, called a structure function, represents a linear combination of up to seven independent variables, comprised of layer thickness-values and the derived coefficients which meet the requirements of the stepwise least squares technique. The thickness variables Z_k, were generated within the BIMED 02R program by utilization of the so-called transgeneration technique

which takes the difference of successive tabulated heights in a sequential manner from the surface to 70 mb. This procedure produced twenty-two independent thickness variables which are directly related to the relevant mean virtual temperature in the layer (P_k, P_{k+1}), by the hypsometric equation. These thickness predictors proved to be highly useful in the specification of T_M.

In the specification of the dependent variable T_M, a statement is permitted as to the maximum number of independent variables to be tested for admission to the regression equation. In the tests considered here, an upper limit of eight independent variables was found in general to give optimal specification of each of the dependent variables. Still another convenient feature of the BIMED 02R program is that the forced entry into the regression of the temperature value being tested as an extreme, the so-called forcing-level temperature, is allowed even though its contribution to the explained variance may become small at the final step in the regression analysis.

Tables 4 through 10 list the variables selected and coefficients of Eq. (2) for each station extreme level considered. The listed structure functions X_M are often highly dependent upon the thickness variables nearly adjacent to the level represented by T_M. However, diversity of thickness-variable selection does occur.

TABLE 4. Full-sample regression equations, with associated thickness-variables at Hall Beach, NWT, for the specification of the January temperatures at indicated levels using $T_J(700)$ as the forcing-level temperature.

$$T_1 = -268.4072 + 0.1473T_3 + 1.0\ X_1$$

$$X_1 = 0.555Z_1 + 0.532Z_2 - 0.521Z_3 + 0.061Z_5 - 0.035Z_{15} + 0.081Z_{16} - 0.059Z_{18}$$

$$T_2 = -285.3345 - 0.0541T_3 + 1.0\ X_2$$

$$X_2 = -0.014Z_1 - 0.016Z_2 + 0.319Z_3 + 0.324Z_4 - 0.020Z_7 + 0.008Z_9$$

$$T_4 = -293.3438 - 0.0755T_3 + 1.0\ X_4$$

$$X_4 = 0.106Z_9 + 0.109Z_{10} + 0.101Z_{11} + 0.095Z_{12} - 0.036Z_{13}$$

$$T_5 = -277.0920 + 0.0092T_3 + 1.0\ X_5$$

$$X_5 = -0.041Z_{13} + 0.171Z_{14} + 0.094Z_{15} - 0.014Z_{16}$$

$$T_6 = -244.0715 + 0.0905T_3 + 1.0\ X_6$$

$$X_6 = -0.026Z_4 - 0.017Z_7 - 0.015Z_{15} + 0.059Z_{16} + 0.201Z_{17} - 0.026Z_{18}$$

$$T_7 = -262.7598 + 0.0306T_3 + 1.0\ X_7$$

$$X_7 = -0.020Z_4 - 0.037Z_{17} + 0.145Z_{18} + 0.114Z_{19} - 0.018Z_{20}$$

$$T_8 = -272.9868 + 0.0065T_3 + 1.0\ X_8$$

$$X_8 = -0.003Z_{12} - 0.026Z_{19} + 0.099Z_{20} + 0.105Z_{21} - 0.047Z_{22}$$

$$T_9 = -254.3952 + 0.0447T_3 + 1.0\ X_9$$

$$X_9 = -0.022Z_4 - 0.008Z_9 - 0.005Z_{15} - 0.029Z_{21} + 0.308Z_{22}$$

X_1 is specified in terms of heights z as

$$X_1 = 0.555(z_{950}-z_{1000}) + 0.532(z_{900}-z_{950}) - 0.521(z_{850}-z_{900}) + 0.061(z_{750}-z_{800}) - 0.035(z_{250}-z_{300}) + 0.081(z_{200}-z_{250}) - 0.059(z_{150}-z_{175}).$$

TABLE 5. Full-sample regression equations, with associated thickness-variables at Resolute, NWT, for the specification of the January temperatures at indicated levels using $T_J(500)$ as the forcing-level temperature.

$$T_1 = -318.9878 - 0.0034T_4 + 1.0\ X_1$$

$$X_1 = 0.410Z_1 + 0.426Z_2 - 0.392Z_3 + 0.233Z_4 + 0.010Z_{15} + 0.068Z_{18} -0.056Z_{22}$$

$$T_2 = -269.0266 + 0.0200T_4 + 1.0\ X_2$$

$$X_2 = -0.022Z_1 - 0.009Z_2 + 0.325Z_3 + 0.284Z_4 - 0.008Z_6$$

$$T_3 = -289.0706 = 0.0791T_4 + 1.0\ X_3$$

$$X_3 = -0.090Z_4 + 0.169Z_5 + 0.133Z_7 + 0.124Z_8 + 0.170Z_6 - 0.011Z_9$$

$$T_5 = -262.3394 + 0.0421T_4 + 1.0\ X_5$$

$$X_5 = -0.007Z_1 - 0.011Z_{12} - 0.034Z_{13} + 0.159Z_{14} + 0.100Z_{15} - 0.017Z_{16}$$

$$T_6 = -272.8584 + 0.0232T_4 + 1.0\ X_6$$

$$X_6 = -0.011Z_{15} + 0.064Z_{16} + 0.206Z_{17} - 0.035Z_{18}$$

$$T_7 = -264.4431 + 0.0184T_4 + 1.0\ X_7$$

$$X_7 = -0.011Z_{12} - 0.032Z_{17} + 0.133Z_{18} + 0.122Z_{19} - 0.018Z_{20}$$

$$T_8 = -270.4817 + 0.0114T_4 + 1.0\ X_8$$

$$X_8 = -0.005Z_7 + 0.020Z_{19} + 0.083Z_{20} + 0.121Z_{21} - 0.058Z_{22}$$

$$T_9 = -269.8740 - 0.0072T_4 + 1.0\ X_9$$

$$X_9 = -0.005Z_2 + 0.004Z_{14} - 0.005Z_{16} - 0.040Z_{21} + 0.325Z_{22}$$

TABLE 6. Full-sample regression equations, with associated thickness-variables at Thule, Greenland, for the specification of the January temperatures at indicated levels using $T_J(300)$ as the forcing-level temperature.

$$T_1 = -257.2419 - 0.0353T_5 + 1.0\ X_1$$

$$X_1 = 0.534Z_1 + 0.554Z_2 - 0.364Z_3 - 0.037Z_9 - 0.041Z_{10} - 0.022Z_{16} + 0.043Z_{18}$$

$$T_2 = -272.4780 + 0.0045T_5 + 1.0\ X_2$$

$$X_2 = -0.019Z_2 + 0.338Z_3 + 0.315Z_4 - 0.020Z_5 - 0.030Z_6$$

$$T_3 = -272.8140 + 0.0093T_5 + 1.0\ X_3$$

$$X_3 = 0.057Z_3 - 0.118Z_4 + 0.166Z_5 + 0.162Z_6 + 0.108Z_7 + 0.102Z_8$$

$$T_4 = -257.5325 + 0.0588T_5 + 1.0\ X_4$$

$$X_4 = 0.074Z_9 + 0.073Z_{10} + 0.119Z_{11} + 0.125Z_{12} - 0.042Z_{13} - 0.019Z_{14}$$

$$T_6 = -246.8040 + 0.0773T_5 + 1.0\ X_6$$

$$X_6 = -0.008Z_1 - 0.035Z_{15} + 0.059Z_{16} + 0.198Z_{17} - 0.013Z_{18}$$

$$T_7 = -270.2495 + 0.0083T_5 + 1.0\ X_7$$

$$X_7 = -0.009Z_1 + 0.006Z_7 - 0.038Z_{17} + 0.156Z_{18} + 0.081Z_{19}$$

$$T_8 = -275.9387 + 0.0118T_5 + 1.0\ X_8$$

$$X_8 = -0.006Z_2 + 0.012Z_4 + 0.061Z_{20} + 0.114Z_{21} - 0.036Z_{22}$$

$$T_9 = -283.4604 - 0.0250T_5 + 1.0\ X_9$$

$$X_9 = -0.006Z_1 + 0.011Z_{10} + 0.012Z_{16} - 0.027Z_{19} + 0.277Z_{22}$$

TABLE 7. Full-sample regression equations, with associated thickness-variables at Thule, Greenland, for the specification of the January temperatures at indicated levels using $T_J(200)$ as the forcing-level temperature.

$T_1 = -331.3064 - 0.2659T_6 + 1.0\ X_1$

$X_1 = \ 0.526Z_1 + 0.543Z_2 - 0.346Z_3 - 0.035Z_9 - 0.039Z_{10} + 0.086Z_{18}$
$\quad -0.156Z_{22}$

$T_2 = -272.2056 - 0.0038T_6 + 1.0\ X_2$

$X_2 = -0.019Z_2 + 0.337Z_3 + 0.314Z_4 - 0.020Z_5 - 0.029Z_6$

$T_3 = -273.0007 + 0.0049T_6 + 1.0\ X_3$

$X_3 = \ 0.057Z_3 - 0.118Z_4 + 0.165Z_5 + 0.162Z_6 + 0.109Z_7 + 0.102Z_8$

$T_4 = -269.6765 + 0.0154T_6 + 1.0\ X_4$

$X_4 = \ 0.074Z_9 + 0.074Z_{10} + 0.118Z_{11} + 0.124Z_{12} - 0.045Z_{13} - 0.006Z_{14}$

$T_5 = -264.6812 + 0.0508T_6 + 1.0\ X_5$

$X_5 = \ 0.009Z_6 - 0.055Z_{13} + 0.179Z_{14} + 0.091Z_{15} - 0.019Z_{16}$

$T_7 = -195.1660 + 0.2897T_6 + 1.0\ X_7$

$X_7 = \ 0.010Z_{15} - 0.021Z_{16} - 0.091Z_{17} + 0.161Z_{18} + 0.081Z_{19}$

$T_8 = -283.8765 - 0.0262T_6 + 1.0\ X_8$

$X_8 = \ 0.006Z_1 + 0.008Z_{19} + 0.060Z_{20} + 0.114Z_{21} - 0.036Z_{22}$

$T_9 = -280.3738 - 0.0026T_6 + 1.0\ X_9$

$X_9 = \ 0.009Z_8 + 0.010Z_{16} - 0.026Z_{19} + 0.277Z_{20}$

TABLE 8. Full-sample regression equations, with associated thickness-variables at Alert, NWT, for the specification of the July temperatures at indicated levels using $T_J(200)$ as the forcing-level temperature.

$$T_1 = -273.6511 - 0.3203T_6 + 1.0\ X_1$$

$$X_1 = 0.425Z_1 + 0.373Z_2 - 0.240Z_3 - 0.033Z_8 - 0.027Z_9 + 0.029Z_{12} + 0.032Z_{16}$$

$$T_2 = -304.2915 - 0.0173T_6 + 1.0\ X_2$$

$$X_2 = 0.026Z_1 + 0.024Z_2 + 0.142Z_3 + 0.399Z_4 - 0.009Z_7 - 0.007Z_{15} + 0.036Z_{18}$$

$$T_3 = -273.5454 - 0.0034T_6 + 1.0\ X_3$$

$$X_3 = 0.034Z_3 - 0.132Z_4 + 0.164Z_5 + 0.172Z_6 + 0.127Z_7 + 0.114Z_8 - 0.007Z_9$$

$$T_4 = -278.9707 + 0.0340T_6 + 1.0\ X_4$$

$$X_4 = 0.006Z_2 + 0.078Z_9 + 0.082Z_{10} + 0.120Z_{11} + 0.112Z_{12} - 0.053Z_{13} + 0.010Z_{14}$$

$$T_5 = -248.0954 + 0.1898T_6 + 1.0\ X_5$$

$$X_5 = -0.011Z_5 + 0.047Z_{11} + 0.032Z_{12} - 0.174Z_{13} + 0.264Z_{14} + 0.082Z_{15} - 0.040Z_{16}$$

$$T_7 = -258.6167 + 0.0495T_6 + 1.0\ X_7$$

$$X_7 = 0.015Z_4 - 0.010Z_{11} - 0.004Z_{15} - 0.047Z_{17} + 0.155Z_{18} + 0.101Z_{19} - 0.014Z_{20}$$

$$T_8 = -272.3201 + 0.0112T_6 + 1.0\ X_8$$

$$X_8 = -0.006Z_2 + 0.010Z_4 - 0.004Z_{14} - 0.004Z_{16} + 0.061Z_{20} + 0.134Z_{21} - 0.063Z_{22}$$

$$T_9 = -273.5842 + 0.0138T_6 + 1.0\ X_9$$

$$X_9 = -0.006Z_1 - 0.010Z_2 + 0.022Z_3 - 0.005Z_{10} + 0.008Z_{13} - 0.013Z_{21} + 0.271Z_{22}$$

TABLE 9. Full-sample regression equation, with associated thickness-variables at Alert, NWT, for the specification of the July temperatures at indicated levels using $T_J(150)$ as the forcing-level temperature.

$$T_1 = -207.5527 - 0.0898T_7 + 1.0\ X_1$$

$$X_1 = 0.424Z_1 + 0.374Z_2 - 0.248Z_3 + 0.024Z_6 - 0.045Z_8 - 0.035Z_9 + 0.022Z_{12}$$

$$T_2 = -329.3796 - 0.1059T_7 + 1.0\ X_2$$

$$X_2 = 0.026Z_1 + 0.024Z_2 + 0.149Z_3 + 0.387Z_4 - 0.008Z_{14} + 0.055Z_{17}$$

$$T_3 = -274.1506 - 0.0104T_7 + 1.0\ X_3$$

$$X_3 = 0.033Z_3 - 0.127Z_4 + 0.160Z_5 + 0.168Z_6 + 0.125Z_7 + 0.112Z_8$$

$$T_4 = -271.5181 + 0.0310T_7 + 1.0\ X_4$$

$$X_4 = 0.080Z_9 + 0.084Z_{10} + 0.112Z_{11} + 0.104Z_{12} - 0.037Z_{13}$$

$$T_5 = -276.9827 + 0.1162T_7 + 1.0\ X_5$$

$$X_5 = 0.047Z_{11} + 0.032Z_{12} - 0.175Z_{13} + 0.264Z_{14} + 0.081Z_{15} - 0.025Z_{16}$$

$$T_6 = -259.7761 + 0.1023T_7 + 1.0\ X_6$$

$$X_6 = 0.007Z_1 + 0.038Z_{16} + 0.184Z_{17} - 0.023Z_{19} + 0.016Z_{21}$$

$$T_8 = -285.4023 - 0.0316T_7 + 1.0\ X_8$$

$$X_8 = -0.005Z_2 + 0.010Z_4 - 0.002Z_{15} + 0.062Z_{20} + 0.136Z_{21} - 0.062Z_{22}$$

$$T_9 = -273.5064 + 0.0135T_7 + 1.0\ X_9$$

$$X_9 = -0.007Z_2 + 0.013Z_3 - 0.004Z_{10} + 0.007Z_{13} - 0.012Z_{21} + 0.270Z_{22}$$

TABLE 10. Full-sample regression equation, with associated thickness-variables at Thule, Greenland, for the specification of the July temperatures at indicated levels using $T_J(100)$ as the forcing-level temperature.

$$T_1 = -215.4731 + 0.3238T_8 + 1.0\ X_1$$

$$X_1 = 0.360Z_1 + 0.374Z_2 - 0.075Z_3 - 0.034Z_4 - 0.041Z_6 + 0.020Z_{15} - 0.021Z_{16}$$

$$T_2 = -303.4197 - 0.0272T_8 + 1.0\ X_8$$

$$X_2 = 0.047Z_1 + 0.048Z_2 + 0.105Z_3 + 0.401Z_4 - 0.012Z_5 - 0.004Z_8 + 0.018Z_{21}$$

$$T_3 = -319.8106 - 0.1370T_8 + 1.0\ X_3$$

$$X_3 = -0.081Z_4 + 0.164Z_5 + 0.156Z_6 + 0.111Z_7 + 0.116Z_8 + 0.047Z_{21} - 0.031Z_{22}$$

$$T_4 = -276.6890 + 0.0313T_8 + 1.0\ X_4$$

$$X_4 = 0.011Z_2 + 0.092Z_9 + 0.094Z_{10} + 0.096Z_{11} + 0.096Z_{12} - 0.033Z_{13}$$

$$T_5 = -266.3704 + 0.0764T_8 + 1.0\ X_5$$

$$X_5 = 0.014Z_7 - 0.073Z_{13} + 0.214Z_{14} + 0.070Z_{15} - 0.016Z_{16}$$

$$T_6 = -255.2715 + 0.1766T_8 + 1.0\ X_6$$

$$X_6 = 0.019Z_{14} - 0.035Z_{15} + 0.061Z_{16} + 0.123Z_{17} + 0.103Z_{18} - 0.044Z_{20}$$

$$T_7 = -184.3870 + 0.0909T_8 + 1.0\ X_7$$

$$X_7 = -0.030Z_6 - 0.050Z_9 + 0.031Z_{11} - 0.014Z_{16} + 0.068Z_{17} + 0.008Z_{18} + 0.021Z_{19}$$

$$T_9 = -257.1123 - 0.020T_8 + 1.0\ X_9$$

$$X_9 = 0.011Z_4 + 0.002Z_{11} - 0.008Z_{13} - 0.003Z_{14} + 0.241Z_{22}$$

B. RELATED STATISTICAL PARAMETERS

Available as outputs from the BIMED 02R program are several related statistical parameters such as:

a. multiple R_r after r steps

b. standard error of estimate, S.E.

c. mean value, $\overline{T}$

d. standard deviation, σ

e. F-statistic after r steps, F_r

f. percent reduced variance, R_r^2.

The significance of R_r^2 in relation to S.E. is given by (Crow et al., 1955) as

$$(S.E.)^2 = \sigma^2((N-1)/(N-r-1))(1-R^2)$$

where

$$\sigma^2 = \sum_{i=1}^{N} (T_i-\overline{T})^2 / N. \qquad (3)$$

It should be noted that S.E. is a measure of the error in specification of T_M after application of the regression equation estimator of form (2).

In (3), σ^2 is the variance of the temperature sample and

N is the sample size

i is the sample-element index

r is the number of predictors selected

R_r is the multiple correlation coefficient after r steps. It follows that the percent unexplained variance $(1 - R^2)$, for sample sizes of 213 to 244, as contained here, is closely approximated by

$$(S.E. / \sigma)^2 = 1 - R^2.$$

The percent explained variance R^2, also called the percent reduced variance, is then given by

$$R^2 = 1 - (S.E. / \sigma)^2. \qquad (4)$$

The mean value of R^2, found in the prediction of the 1000 mb temperatures using the full-data set, was 0.8478 and 0.7480 with the 10% extreme-data set. For all levels above 1000 mb, the pooled mean R^2 for the full-data set was 0.9672 and 0.9596 for the 10% extreme-data set. The scores at 1000 mb are somewhat lower than those of the upper levels and this is to be expected, in that the surface variabilities bring periodic and random-effect terms into the specification of the temperature at 1000 mb.

TABLE 11. Regression statistics at mandatory-pressure levels at Hall Beach, NWT, using T(700) as the forcing level. Part (a) refers to the full-data January sample; part (b) refers to the nominal 10% cold extreme sample of T(700); part (c) lists the sounding level mean temperatures corresponding to the 1% extreme cold set of T(700).

Level	(a) N = 243 cases				(b) N = 26 cases				(c) N = 5
mb	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C
1000	-26.11	7.602	.9449	2.5314	-35.04	3.893	.8430	2.1832	-35.24 D
850	-22.63	7.238	.9984	.4185	-35.60	3.295	.9953	.3322	-39.52 D
700	-27.35	6.467			-39.68	3.186			-44.96 D
500	-39.93	4.994	.9963	.4350	-45.98	2.847	.9843	.5232	-44.04 M
300	-55.09	5.578	.9963	.4928	-45.45	4.965	.9962	.4515	-41.02 D
200	-53.53	6.071	.9966	.5112	-45.88	5.279	.9963	.4718	-42.96 D
150	-53.71	6.517	.9982	.3916	-47.40	6.796	.9981	.4400	-45.20 D
100	-55.49	7.689	.9988	.3816	-51.11	7.847	.9988	.3816	-48.64 D
70	-57.38	8.942	.9987	.4624	-54.41	8.936	.9983	.5410	-50.28 D

TABLE 12. Regression statistics at mandatory levels at Resolute, NWT, using T(500) as the forcing-level temperature. Part (a) refers to the full-data January sample; part (b) refers to the nominal 10% cold extreme sample of T(500); part (c) lists the sounding-level mean temperatures corresponding to the 1% extreme cold set of T(500).

Level mb	(a) N = 237 cases				(b) N = 25 cases				(c) N = 3
	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C
1000	-29.10	6.349	.9061	2.7325	-35.52	4.273	.8057	2.6435	-39.00 D
850	-24.56	5.631	.9961	.5010	-31.18	3.457	.9952	.3533	-35.50 D
700	-28.87	4.856	.9948	.5041	-35.71	3.175	.9957	.3078	-40.37 D
500	-41.92	4.043			-48.68	1.573			-51.73 D
300	-55.85	5.225	.9949	.5341	-57.26	5.916	.9978	.4073	-54.03 M
200	-53.89	6.121	.9959	.5576	-56.04	6.043	.9978	.4230	-55.20 M
150	-54.56	7.036	.9987	.3686	-58.68	5.517	.9989	.2679	-59.37 D
100	-56.48	8.510	.9991	.3925	-61.95	6.982	.9997	.2663	-65.70 D
70	-58.34	7.578	.9992	.3925	-64.48	7.541	.9994	.2663	-70.77 D

TABLE 13. Regression statistics at mandatory-pressure levels at Thule, Greenland, using T(300) as the forcing-level January temperature. Part (a) refers to the full-data sample; part (b) refers to the nominal 10% cold extreme sample of T(300); part (c) lists the sounding-level mean temperatures corresponding to the 1% extreme cold set of T(300).

Level	(a) N = 213 cases				(b) N = 13 cases				(c) N = 2
mb	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C
1000	-20.50	7.097	.9629	1.9528	-21.41	5.931	.8766	3.1272	-25.20 D
850	-21.91	7.662	.9975	.5504	-22.55	6.079	.9973	.4902	-24.83 D
700	-27.04	6.187	.9956	.5918	-28.68	6.095	.9989	.3166	-29.73 D
500	-40.28	4.758	.9949	.4902	-42.41	3.887	.9975	.2996	-42.90 D
300	-55.58	5.049			-63.03	.880			-64.47 D
200	-54.86	6.791	.9966	.5673	-61.88	2.375	.9877	.4067	-62.83 D
150	-55.22	7.216	.9977	.4999	-63.02	2.809	.9972	.2317	-63.73 D
100	-57.10	9.084	.9988	.5461	-67.51	2.493	.9834	.4949	-67.93 D
70	-59.12	10.804	.9988	.5461	-70.57	3.197	.9869	.5642	-71.50 D

TABLE 14. Regression statistics at mandatory-pressure levels at Thule, Greenland, using T(200) as the forcing-level January temperature. Part (a) refers to the full-data sample; part (b) refers to the nominal 10% cold extreme sample of T(200); part (c) lists the sounding-level mean temperatures corresponding to the 1% extreme cold set of T(200).

Level mb	(a) N = 213 cases				(b) N = 18 cases				(c) N = 3
	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C
1000	-20.50	7.097	.9636	1.9347	-12.83	7.471	.9806	1.5586	-9.87 D
850	-21.91	7.622	.9975	.5503	-13.18	6.947	.9986	.3974	-5.30 D
700	-27.04	6.187	.9956	.5926	-20.51	5.885	.9983	.3622	-12.67 D
500	-40.28	4.758	.9949	.4885	-35.74	5.012	.9989	.2516	-29.97 D
300	-55.58	5.049	.9945	.5355	-59.37	2.170	.9900	.3255	-57.03 M
200	-54.86	6.791			-67.93	3.750			-74.70
150	-55.22	7.216	.9979	.4760	-66.57	4.252	.9859	.7578	-73.57 D
100	-57.10	9.084	.9985	.5091	-70.32	3.135	.9915	.4347	-73.43 D
70	-59.12	10.804	.9987	.5505	-74.05	3.804	.9971	.3096	-77.50 D

TABLE 15. Regression statistics at mandatory-pressure levels at Alert, NWT, using T(200) as the forcing-level July temperature. Part (a) refers to the full-data sample; part (b) refers to the nominal 10% cold extreme sample of T(200); part (c) lists the sounding-level mean temperatures corresponding to the 1% extreme cold set of T(200).

	(a) N = 236 cases				(b) N = 18 cases				(c) N = 3
Level mb	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C
1000	3.26	3.673	.9034	1.6014	1.30	1.317	.9107	1.6014	0.05 D
850	-0.91	3.203	.9865	.5341	-4.40	2.363	.9936	.2848	-6.65 D
700	-9.02	3.396	.9866	.5638	-13.43	2.296	.9833	.4444	-16.15 D
500	-23.97	3.211	.9932	.3809	-28.11	2.435	.9933	.3002	-31.30 D
300	-47.59	2.545	.9654	.6747	-44.93	3.775	.9823	.7529	-40.05 D
200	-42.19	2.789			-38.13	.644			-36.70
150	-42.04	1.779	.9745	.4060	-39.58	.714	.8730	.3707	-38.90 D
100	-41.87	1.263	.9735	.2935	-40.58	.867	.9684	.2301	-40.95 M
70	-40.87	1.104	.9670	.2862	-39.64	1.526	.9883	.2479	-40.25 M

TABLE 16. Regression statistics at mandatory-pressure levels at Alert, NWT, using T(150) as the forcing-level July temperature. Part (a) refers to the full-data sample; part (b) refers to the nominal 10% cold extreme sample of T(150); part (c) lists the sounding-level mean temperatures corresponding to the 1% extreme cold set of T(150).

Level	(a) N = 236 cases				(b) N = 22 cases				(c) N = 2
mb	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C
1000	3.26	3.673	.9017	1.6150	2.19	2.434	.8643	1.2874	0.00 D
850	-0.91	3.203	.9866	.5316	-3.54	2.320	.9939	.2684	-4.65 D
700	-9.02	3.396	.9866	.5638	-12.25	2.644	.9808	.5426	-13.75 D
500	-23.97	3.211	.9929	.3863	-26.43	2.209	.9866	.3791	-26.25 M
300	-47.59	2.545	.9638	.6886	-47.16	3.210	.9755	.7428	-43.50 D
200	-42.19	2.789	.9802	.5593	-38.93	.939	.9242	.3768	-38.10 D
150	-42.04	1.779			-39.24	.537			-38.20
100	-41.87	1.263	.9735	.2934	-40.27	.498	.8944	.2341	-40.60 M
70	-40.87	1.104	.9664	.2881	-39.59	1.325	.9875	.2194	-39.45 D

TABLE 17. Regression statistics at mandatory-pressure levels at Thule, Greenland, using T(100) as the forcing-level July temperature. Part (a) refers to the full-data sample; part (b) refers to the nominal 10% cold extreme sample of T(100); part (c) lists the sounding-level mean temperatures corresponding to the 1% extreme cold set of T(100).

Level	(a) N = 236 cases				(b) N = 23 cases				(c) N = 2
mb	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C	Std. Dev. °C	Mult. Corr. Coef.	Std. Err. of Est. °C	Mean °C
1000	3.70	3.199	.8578	1.6732	1.86	1.891	.7549	1.3009	0.85 D
850	-0.73	3.033	.9741	.6945	-3.42	2.364	.9066	1.0464	-4.90 D
700	-7.76	3.147	.9733	.7320	-11.16	3.480	.9645	.9642	-10.75 M
500	-23.05	3.097	.9863	.5190	-26.27	3.363	.9871	.5641	-25.25 M
300	-47.54	2.399	.9569	.7056	-46.79	2.669	.9598	.7854	-45.40 D
200	-44.17	3.778	.9748	.8563	-39.82	1.541	.9643	.4278	-39.80 D
150	-43.57	2.770	.6991	2.0151	-40.59	1.112	.8382	.6361	-40.00 D
100	-43.20	1.429			-40.59	.409			-40.10 D
70	-42.08	1.237	.9656	.3257	-40.53	.674	.9638	.1886	-39.95 D

IV. ANALYTICAL PROCEDURE USING THE REGRESSION TECHNIQUE

A. RESULTS OF THE REGRESSIONS

Part (a) of Tables 11 through 17 lists the mean, standard deviation, multiple correlation coefficient and standard error of estimate for the full-data sample of each station and at all mandatory levels except the forcing level. These statistics were computed after application of the appropriate regression equations listed in Tables 4 through 10. Part (b) of Tables 11 through 17 gives similar results, after application of the identical regression equations used in part (a), to a regression analysis of the data set which contains only the "10% extreme" set of forcing-level sample atmospheres. This means that $T(P_J)$ was chosen if it lay within the particular 10% Gaussian distribution extreme under consideration. The remainder of the mandatory levels of the extreme sounding, $T_M(.10)$, were then subjected to the same regression analysis as the full-data samples of part (a). This nominal set of extreme atmospheres was selected by choosing those soundings for which $T(P_J) \geq T_J(.10)$ for the warm extremes, and $T(P_J) \leq T_J(.10)$ for the cold extremes. These extreme 10% values $T_J(.10)$ are defined by the Gaussian distribution limits listed in Eq. (1).

Part (c) lists the mean temperature profile defined at mandatory levels which correspond to the set of soundings having $T(P_J)$ at the 1% extreme. There were two ways of selecting the 1% extreme cases: (i) application of an a priori test to determine the values $T(P_J)$ which fell within the required range indicated by the Gaussian distribution; (ii) if there were no values of $T(P_J)$ fitting this constraint, then either the two or three coldest (for cold extremes) or warmest values,

were chosen a posteriori from the regression-determined 1% set as representing the set of soundings corresponding to the 1% extreme $T(P_J)$-values. The corresponding set of $T(P_M)$ values were then simply averaged and considered to represent the expected vertical profile having $T(P_J)$ in the 1% extreme.

B. THE TEST FOR SIGNIFICANT SOUNDING DIFFERENCES

In view of the small sample sizes in parts (b) and (c) of Tables 11,..., 17, it was not feasible to test for significant difference by random sampling of the 10% set relative to the 1% set of $T(P_J)$ generated soundings. It was decided therefore to test for significant differences in defining the $\overline{T}_M$ values predicted in each part (b) of Tables 11,..., 17 against those $\overline{T}_M$ values of part (a) obtained by the identical prediction process, but using the full-data sample: that is, in case (a) there were no a priori limiting conditions set up regarding a particular value of $T(P_J)$.

In order to check the latter hypothesis, a Student's t-test was performed to test whether the 10% samples of prediction means of part (b) were significantly different from those of the full-data sample means of part (a). The pooled t-statistic relating to the $\overline{T}_M$ prediction differences of the part (a) and part (b) means may be formulated after Brooks and Carruthers (1953) with (N+n-(r+2)) degrees of freedom, after computing the two sets of predictive mean values and their standard errors. The t-statistic is formulated as follows:

$$t(N+n-(r+2)) = (\overline{T}_{FULL}-\overline{T}_{.10}) \Bigg/ \left[\left(\frac{(N-9)\sigma_E^2 + (n-2)\sigma_{E,.10}^2}{(N+n-r-2)}\right)^{\frac{1}{2}}\left(\frac{1}{N} + \frac{1}{n}\right)^{\frac{1}{2}}\right] \qquad (5)$$

Here the $\overline{T}$'s are the part (a) and the part (b) sample temperature means $\overline{T}_M$ (at the mandatory levels) based upon identical prediction equations. σ_E is the part (a) standard error of estimate, and $\sigma_{E,.10}$ is the part (b) standard error of estimate. The number of degrees of freedom associated with the two σ_E values are (N-r-1) and (n-2), respectively, in parts (a) and (b) with

N = the full-sample size

n = the nominal 10% extreme-sample size

r = the number of predictors, r = 8.

The pooled t-statistic may be computed from the horizontally adjacent sections of Tables 11,..., 17 and with the proper number of (N+n-(r+2)) degrees of freedom for t. Tables 18 and 19 of Appendix 1 list the resulting values at mandatory levels for each station under consideration.

These t-values were tested against critical $t_{.01}$-values (e.g., Crow et al, 1960) with the same number of degrees of freedom used in (5). This number of degrees of freedom usually fell into the tabular class for t-values corresponding to an infinite number of degrees of freedom. Thus for example $t_{.01}$, with an infinite number of degrees of freedom, is t = 2.326. Tables 18 and 19 show one value below the 99% confidence limit specified by $t_{.01}$ = 2.326. That value, corresponding to the 1000 mb level of Thule (with P_J = 300 mb), still yields a confidence limit of 92%. The random chance of the $\overline{T}_M$ specifications of parts (a) and (b) of Tables 11,..., 17 being drawn from the same population if t = 7.0 is approximately one chance in 5 X 10^{12} and is much smaller for t > 7.0. Most of the t-values in Tables 18 and 19 exceeded t = 7.0 with exceptions principally at P = 1000 mb.

C. THE TYPE-(C) PROFILE CLASSIFICATION

As noted previously, it was not feasible to employ a t-test utilizing the extremely small sample size of part (c) of Tables 11,..., 17. However, it is useful to explain physically the pattern of trends in the sample means $\overline{T}_M$ of part (c) relative to those of (b). The code symbol "D" for "direct", used in part (c), means that $\overline{T}_M(c)$ is displaced directly in the same sense from $\overline{T}_M(b)$ as $\overline{T}_M(b)$ is from $\overline{T}_M(a)$ at the level under consideration. The code symbol "M" for "mixed" means that $|\overline{T}_M(c) - \overline{T}_M(a)| < |\overline{T}_M(b) - \overline{T}_M(a)|$ so that a consistent displacement in the temperature extreme does not occur in progressing across the columns of $\overline{T}_M$-values. This code symbolism was used for all cases considered and is summarized under part (c) of Tables 11,..., 17. To some extent, the nature of these displacements, whether direct or mixed, is indicated also by Fig. 2,..., Fig. 8, which show only the comparison of the $\overline{T}_M(c)$ and $\overline{T}_M(a)$ profiles.

V. RESULTS SUMMARY FOR INDIVIDUAL STATIONS

A. WINTER CASES

This section will consider those cases investigated as cold extremes and summarized in Tables 11,..., 14 and Figs. 2,..., 5.

1. Hall Beach, NWT; $T(P_J) = T_J(700)$

Table 11(a) shows the results of the stepwise regression applied to the specification of the January full-sample temperatures ($T_M = T_1, T_2, T_4, T_5, T_6, T_7, T_8$ and T_9) at the mandatory levels of Hall Beach by an equation of form (2). An eight-predictor equation for T_M, at each level, was generated by use of the BIMED 02R program with the results shown. Very high multiple correlation coefficients were obtained, indicating a functional relationship for T_M at all levels. Table 11(b) lists analagous results for the data sample corresponding to the 10% cold extreme at the forcing level, with $\overline{T}_M(b)$ being determined by the same specification equation used in part (a). Part (c) presents the resultant estimated mandatory-level means $\overline{T}_M(c)$ derived from the five cases which fell into the 1% extreme forcing level subset of part (b). Part (c) then, gives the expected vertical profile corresponding to the 1% extreme at $T(P_J) = T(700)$. It is suggested that the mean $\overline{T}_M(c)$ profile, relative to that of $\overline{T}_M(b)$, should be physically as well as statistically explainable from the conditions imposed by the choices of P_J. At Hall Beach, the latter selection at 700 mb was consistent with the existence of an extreme-cold troposphere, $\overline{T}_M(c) < \overline{T}_M(b)$, and an extreme-warm stratosphere for winter. The only case of a mixed classification M, occurs for $\overline{T}_M(500)$, where according

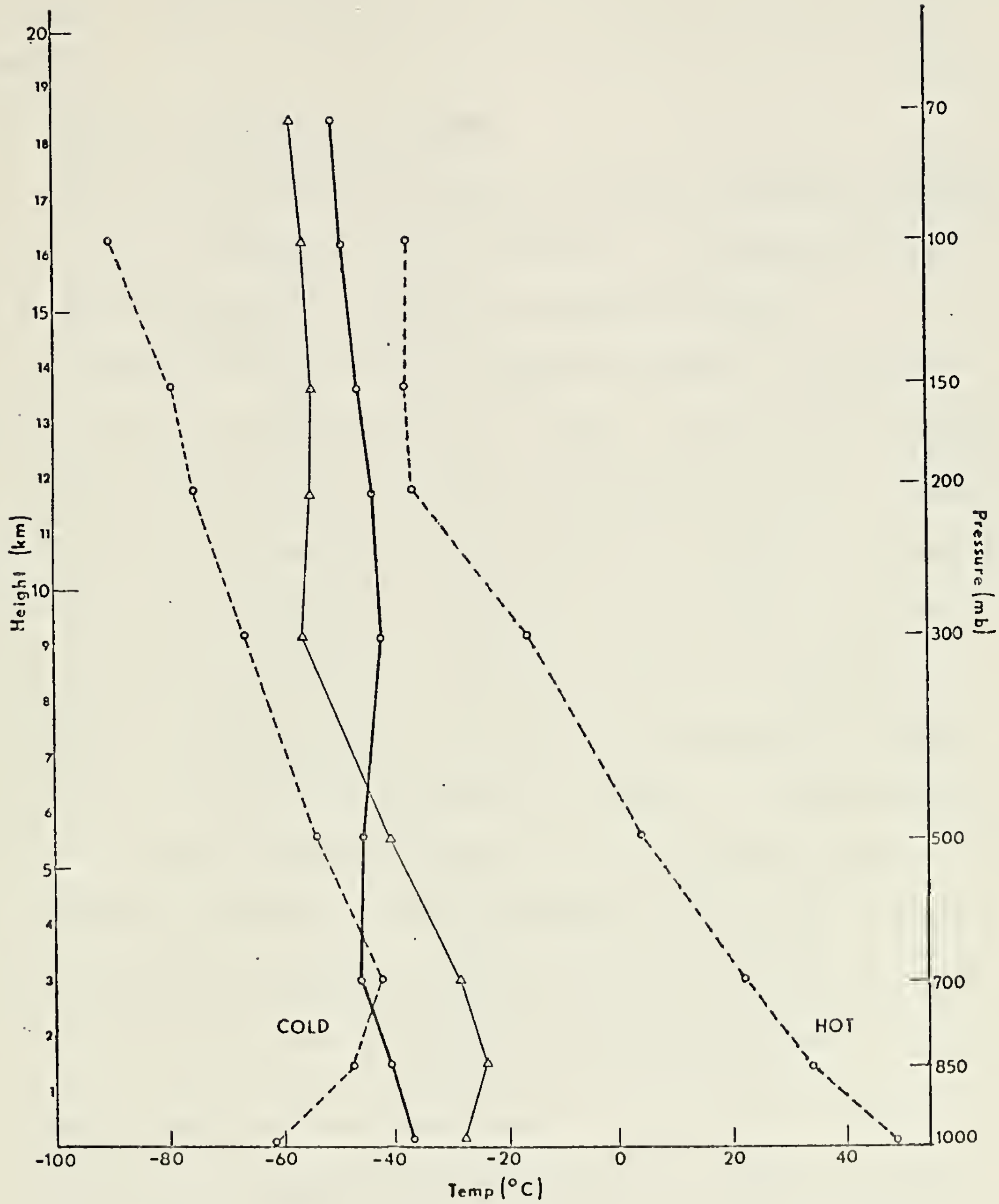

FIG. 2. The heavy solid line shows the mean regression-determined vertical temperature sounding over Hall Beach, NWT, corresponding to the set of 1% cold extreme occurrences of $T_J(700)$. The thin solid line depicts the January mean Hall Beach vertical sounding (1967-70).

to Fig. 2, the crossover between the mean and extreme soundings occurs. The nature of the $\overline{T}_M(c)$ profile is then consistent with the displacement of the $\overline{T}_M(b) - \overline{T}_M(a)$ profiles except at the crossover point near the tropopause.

2. Resolute, NWT; $T(P_J) = T_J(500)$

In the case of Resolute, Table 12(c) shows a somewhat different result from that discussed above for Hall Beach. At Resolute we now start with the premise of an initially specified very cold $\overline{T}_J(c) < T_{.01}(500)$. This level is excluded from being an effective crossover point in the results of Fig. 3 except in the region of P=300 mb where a slight tendency toward a warm-stratosphere does occur. Comparing the two cases, the selection of an extreme cold $T_J(500)$ dictates the presence of a cold stratosphere coupled with a cold troposphere below. For nearly all levels at Resolute, the $\overline{T}_M(c) - \overline{T}_M(a)$ trend bears out that indicated by $\overline{T}_M(b) - \overline{T}_M(a)$ in the sense of maintaining the $\overline{T}_M(c)$ atmosphere relatively cold at virtually all levels. In contrast with the situation at Hall Beach, the Resolute type (c) atmosphere more nearly resembles the case of a cold tropospheric vortex with an overlying stratospheric cold vortex, whereas the Hall Beach type (c) case seems to occur with the warm stratospheric anticyclone aloft.

3. Thule, Greenland; $T(P_J) = T_J(300)$

This case is graphically depicted in Fig. 4, which shows that the *a priori* condition of a 1% cold temperature extreme at P_J=300 mb at Thule is associated with a cold stratosphere as well as a cold troposphere at all levels above and below P_J=300 mb. The extreme vertical profile for this regime ($T(P_J) = T(300)$) at Thule resembles that at Resolute, where there was also evidence of a similar coupling between

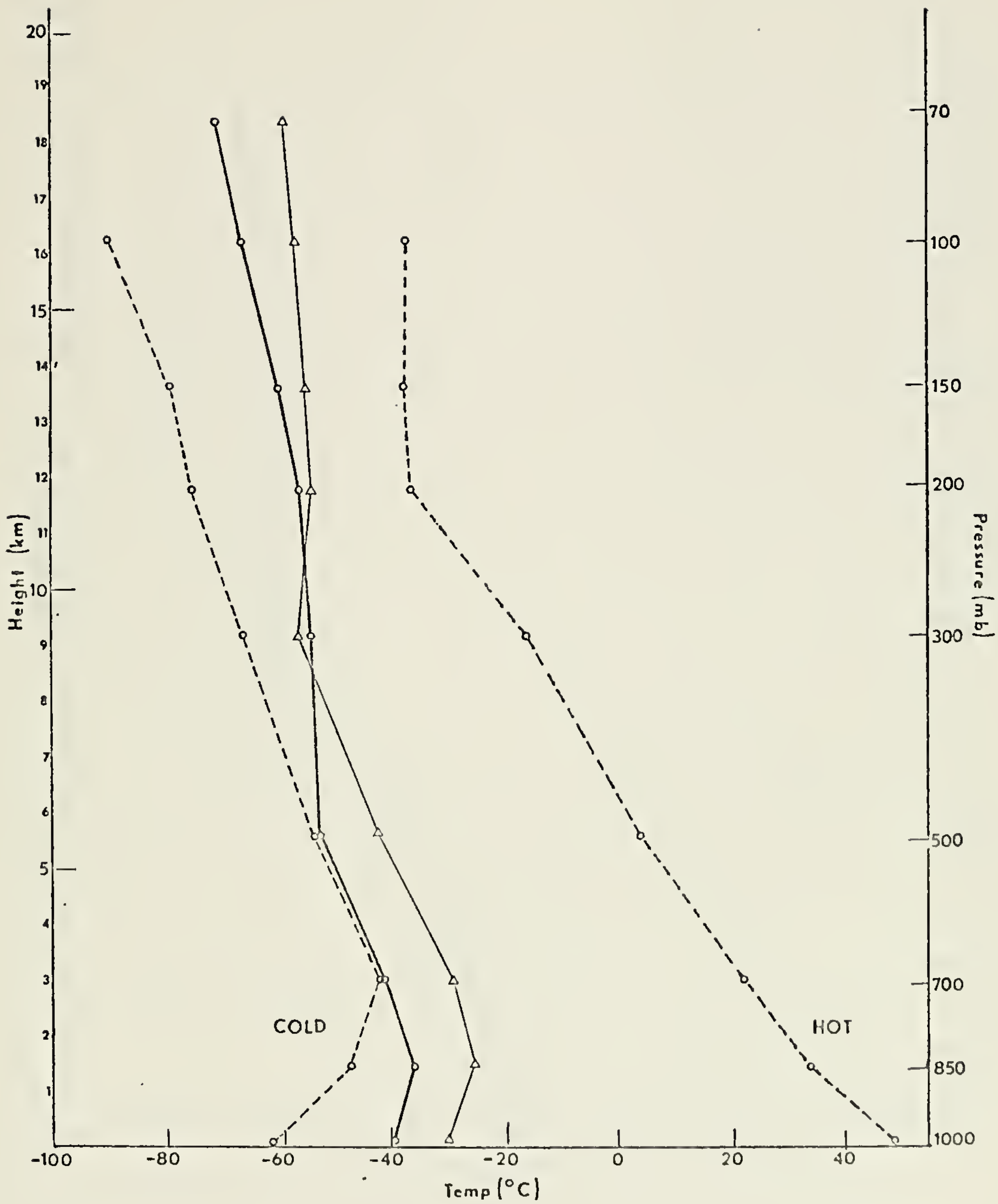

FIG. 3. The heavy solid line shows the mean regression-determined vertical temperature sounding over Resolute, NWT, corresponding to the set of 1% cold extreme occurrences of $T_J(500)$. The thin solid line depicts the January mean Resolute sounding (1967-70).

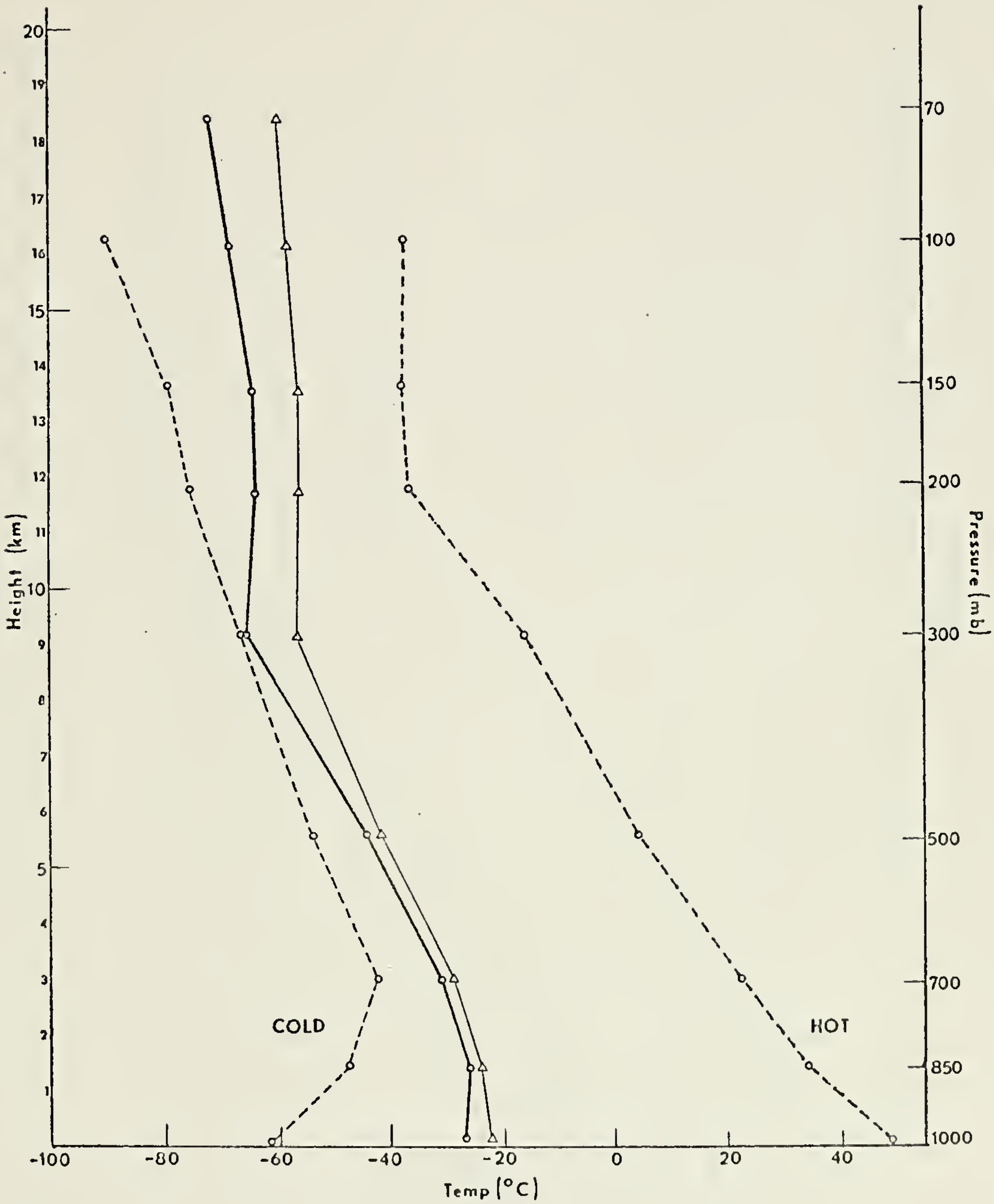

FIG. 4. The heavy solid line shows the mean regression-determined vertical temperature sounding over Thule, Greenland, corresponding to the 1% cold extreme occurrences of $T_J(300)$. The thin solid line depicts the January mean Thule sounding (1967-70).

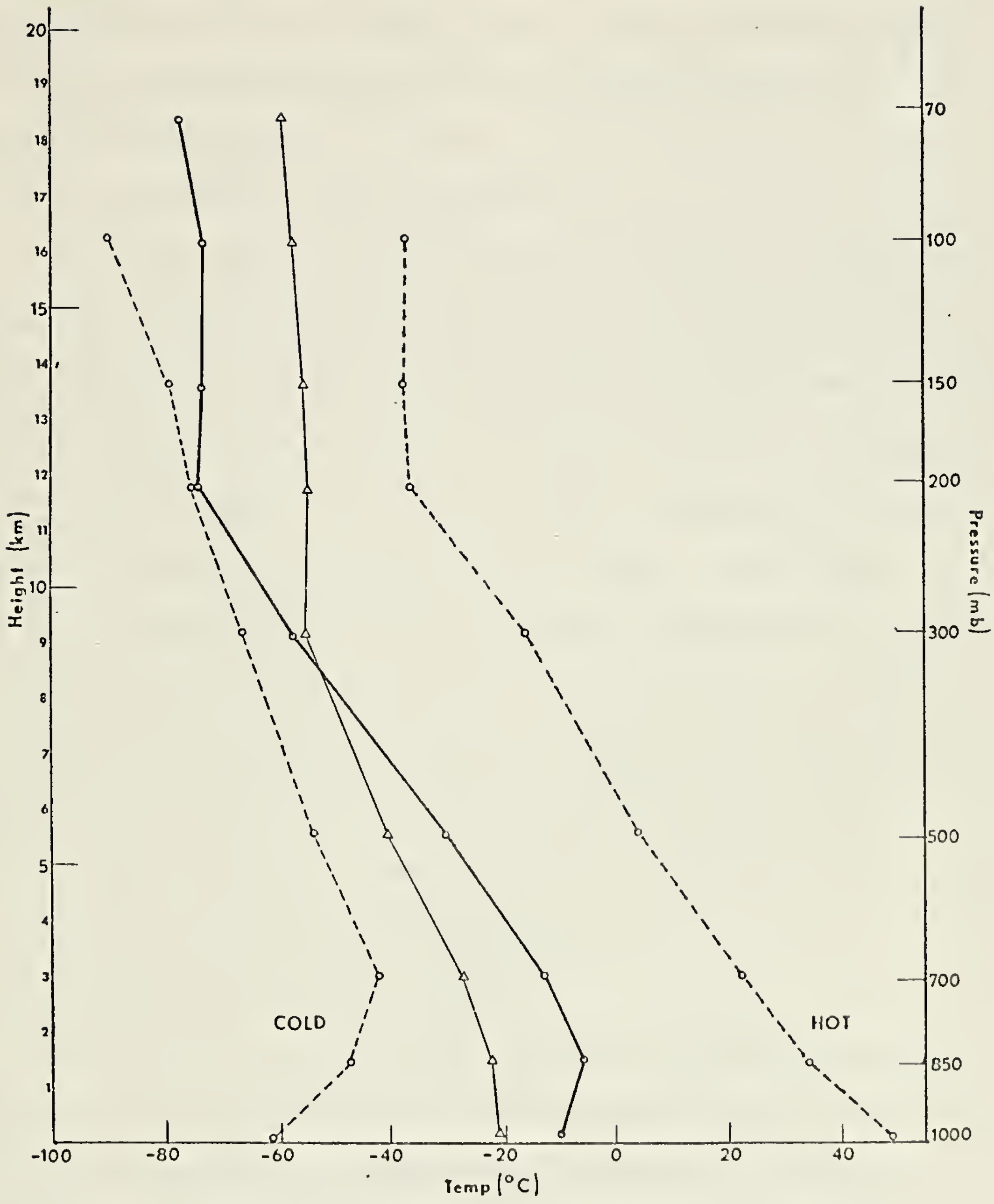

FIG. 5. The heavy solid line shows the mean regression-determined vertical temperature sounding over Thule, Greenland, corresponding to the 1% cold extreme occurrences of $T_J(200)$. The thin solid line depicts the January mean Thule sounding (1967-70).

cold stratospheric temperatures in case (c) and a cold-extreme upper troposphere. Now, however, Fig. 4 shows no crossover point between the mean and extreme profiles. Likewise Table 13 shows the direct transition D-type classification applicable at all levels of the mean temperature profile $\overline{T}_M(c)$.

4. Thule, Greenland; $T(P_J) = T_J(200)$

This case differs from the previous case ($T(P_J) = T_J(300)$), in that the latter was associated by regression development with $\overline{T}_M(200) = -62.83^{\circ}C$, whereas the coldest 1% input temperature $\overline{T}_J(200)$ was $-74.70^{\circ}C$. The imposition of the latter condition into the regression analysis at Thule ($T_J(200)$) is associated with the coldest stratospheric regime encountered during the entire study of this paper. With $\overline{T}_J(200) = -74.70^{\circ}C$, all regression-generated stratospheric temperatures are typical of those in the cold Arctic vortex at and above 200 mb. However Fig. 5, shows that in this case there is a crossover in the $\overline{T}_M(a)$, $\overline{T}_M(c)$ profiles. Furthermore by comparison of Table 14(c) with Tables 11(c), 12(c) and 13(c), it is clear that this case corresponds to the warmest of the four winter tropospheres discussed here.

5. Combined Winter Regimes, Summary

The analysis of the four sets of type (c) winter extremes has indicated that the 300 mb level is a key indicator in the type of atmosphere to be expected in the Canadian Arctic region. In summary, these regimes have been identified by the four stratifications:

(a) Hall Beach $T_J(700)$, cold extreme

(b) Resolute $T_J(500)$, cold extreme

(c) Thule $T_J(300)$, cold extreme

(d) Thule $T_J(200)$, cold extreme

Case (a) corresponds to an abnormally cold troposphere with a resultant abnormally warm stratosphere (Fig. 2). Cases (b) and (c) correspond jointly to cold stratospheres and cold tropospheres (Figs. 3 and 4), but neither cold layer exists at a particular extreme. Finally case (d) is characterized by a record cold stratosphere and a near-record warm lower troposphere. Cases (a) and (d) show negative correlations of temperature across the tropopause, a property documented at mid-latitude stations by Cole and Nee (1965). However cases (b) and (c) have a consistent positive correlation throughout the atmosphere and have yet to be documented fully.

B. SUMMER CASES

This section will consider those cases as having warm extremes at forcing levels and are summarized in Tables 15, 16 and 17 and Figs. 6, 7 and 8. The same stepwise regression procedure was applied to the July data of the warm extreme stations as was applied to the January data of the cold extreme stations. The results are presented below.

1. Alert, NWT; $T(P_J) = T_J(200)$

Table 15 lists the statistical results of the analysis of Alert with $T(P_J) = T_J(200$. Consistently high multiple correlations are shown indicating a high degree of confidence in the predicted $\overline{T}_M$ values including those of part (c). This confidence is reinforced by the consistent direct transition D-type classification applicable at most levels of the mean temperature profile $\overline{T}_M(c)$. Mixed tendencies occur at 100 mb and 70 mb but are characterized by extremely small temperature differentials (less than 0.5°C) in the $\overline{T}_M(c)$ - $\overline{T}_M(b)$ trends. A crossover of the $\overline{T}_M(c)$ extreme and $\overline{T}_M(a)$ profiles exists between 500 mb

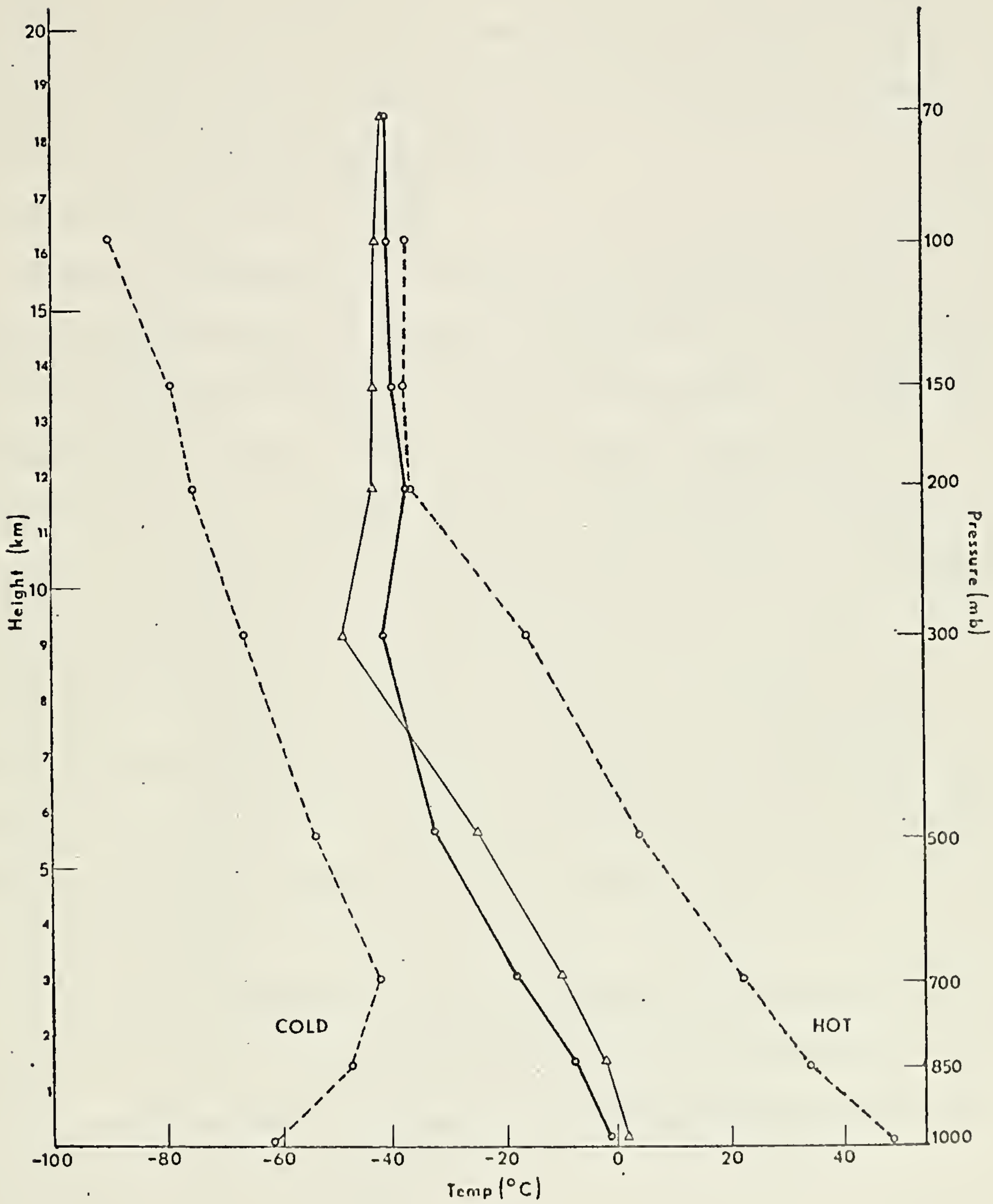

FIG. 6. The heavy solid line shows the mean regression-determined vertical temperature sounding over Alert, NWT, corresponding to the 1% warm extreme occurrence of $T_T(200)$. The thin solid line depicts the July mean Alert sounding (1967-70).

and 300 mb (Fig. 6), indicating an extreme warm stratosphere overlying a relatively cool troposphere. This phenomenon is also characteristic of mid-latitude negative inter-level temperature correlations shown by Cole and Nee (1965) to exist near the level of the tropopause during periods of cyclone-anticyclone variability.

2. Alert, NWT; $T(P_J) = T_J(150)$

Table 16 corresponds to the analysis of Alert with $T(P_J) = T_J(150)$. Very high multiple correlation coefficients in both parts (a) and (b) are again shown yielding a resultant high degree of confidence in the diagnostic procedure of isolating "extreme" atmospheres. Figure 7 indicates that a marked similarity exists between the $\overline{T}_M(a)$ and $\overline{T}_M(c)$ profile pairs and the corresponding pair for Alert of $T_J(200)$. Table 16 indicates that the $\overline{T}_M(c)$ regression-generated atmosphere is displaced primarily in a "direct" sense relative to the $\overline{T}_M(b)$ atmosphere. It should also be noted that the forcing extreme $T_J(150)$ at the nominal 10% and 1% values differ only by about 1^oC. The comparison between $\overline{T}_M(c)$ and $\overline{T}_M(b)$ is generally of the same character as that of $\overline{T}_M(c)$ relative to $\overline{T}_M(a)$, (Fig. 7).

The same conditions of warm stratosphere and relatively cool troposphere, as well as the crossover between 500 mb and 300 mb, suggest a similar comparison with regard to this climatic regime as with the preceding one (Alert $T_J(200)$).

3. Thule, Greenland; $T(P_J) = T_J(100)$

The final table of regression statistics, Table 17, corresponds to the analysis of Thule at $T(P_J) = T_J(100)$. Once again there are very high multiple correlation coefficients in both parts (a) and (b). In addition the atmospheric profiles, both in the mean and 1% extreme

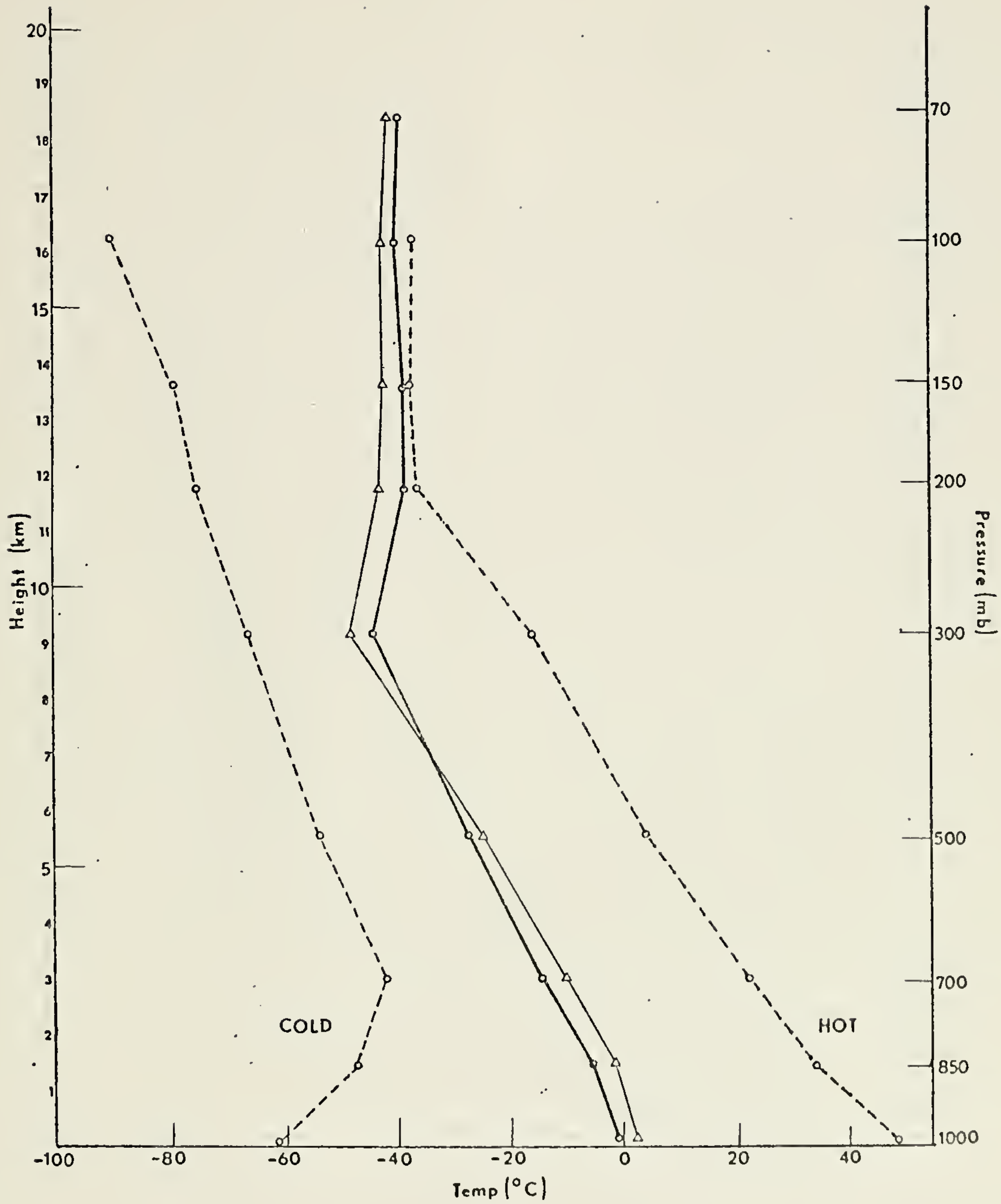

FIG. 7. The heavy solid line shows the mean regression-determined vertical temperature sounding over Alert, NWT, corresponding to the 1% warm extreme occurrence of $T_J(150)$. The thin solid line depicts the July mean Alert sounding (1967-70).

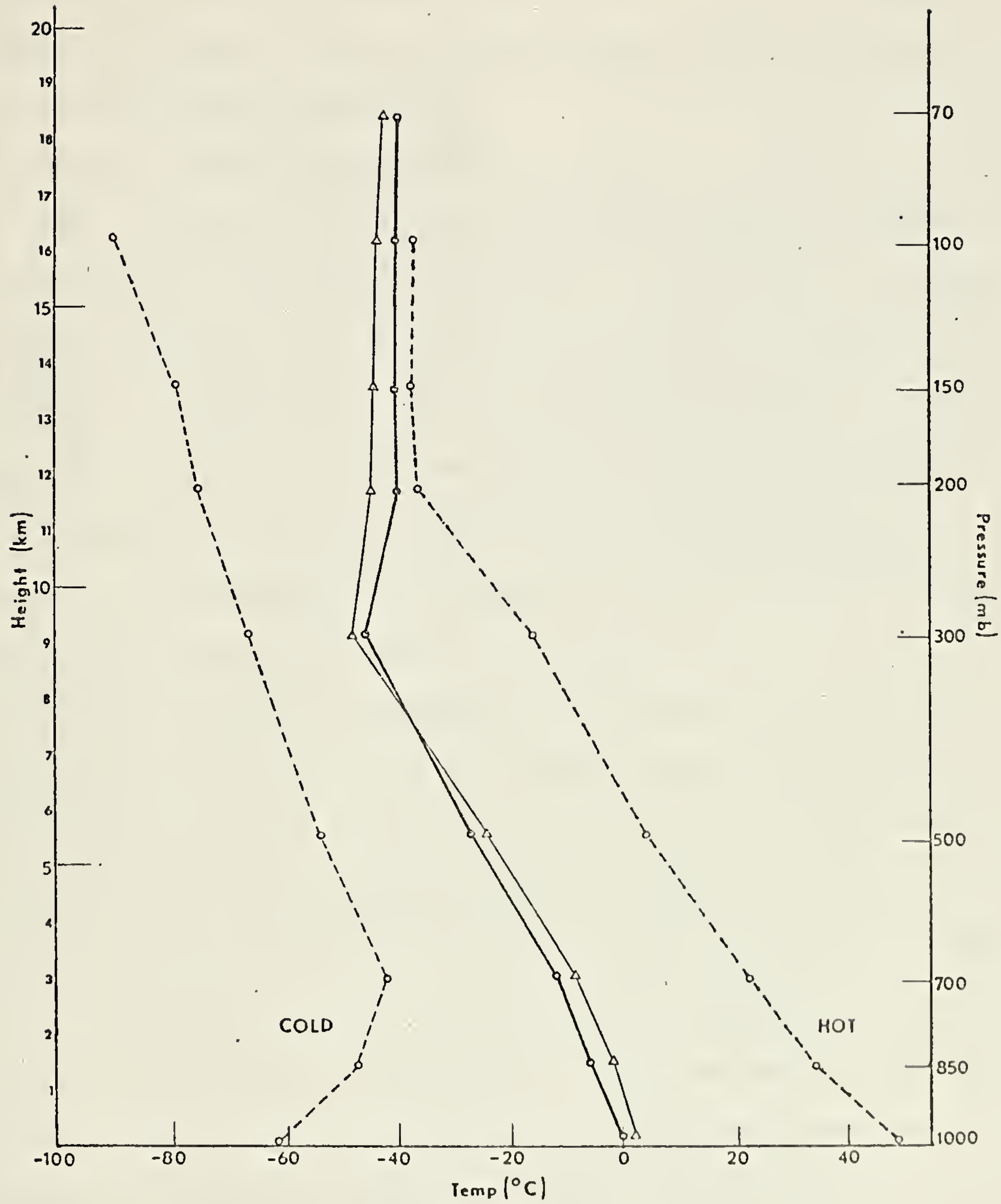

FIG. 8. The heavy solid line shows the mean regression-determined vertical temperature sounding over Thule, Greenland, corresponding to the 1% warm extreme occurrence of $T_J(100)$. The thin solid line depicts the July mean Thule sounding (1967-70).

(Fig. 8), were nearly identical to the corresponding profile pairs in the previous two warm cases (Figs. 6 and 7). This indicates a strong tendency of the synoptic processes acting at Alert in July to also be acting at Thule during the same period.

4. Combined Summer Regimes, Summary

Each of the July cases is based upon the assumption of a warm extreme forcing-level temperature existing at or above 200 mb. In each case, there proved to be a concurrent warm temperature at 300 mb, that is, stratospheric warming above 200 mb has extended downward to 300 mb in July. At tropospheric levels, however, the mean temperatures of parts (b) and (c) are generally colder than the full-sample means. Figures 6, 7 and 8 are graphical presentations of the mean temperatures shown in parts (a) and (c) of Tables 15, 16 and 17.

Three factors should be recognized as being relevant to the July data considered. (i) In two of the three cases, the nominal 1% extreme temperature at the forcing level $T(P_J)$ did not occur, and the class was redefined by a posteriori methods to comprise the set of two warmest observations at the forcing level. (ii) The difference between the parts (b) and (c) mean mandatory level profiles ($\overline{T}_M(b)$ and $\overline{T}_M(c)$), at and above 300 mb, never exceeded 1.0^oC. (iii) The dominant feature of all of the summer case-studies was the consistent occurrence of stratospheric-tropospheric temperature reversals noted in the preceding paragraph of this section.

VI. CONCLUSIONS

The multiple regression procedure presented for development of vertically consistent model atmospheres is statistically dependent upon the input condition of a known extreme temperature at a given flight level. The procedure has been shown to be highly definitive in determining the most probable vertical temperature profile over the stations of interest. Extremely warm stratospheric temperatures are not associated with extremely warm tropospheric temperatures, thus integrated ballistic density anomalies tend to be nearly compensated. Likewise, extremely cold tropospheric temperatures tend to be reversed in the stratosphere. These two points become extremely important in the determination of ballistic parameters. As a result, aerothermodynamic calculations can be made for flight conditions involving changes in altitude by using an atmospheric temperature profile which is consistent with the proposed MIL-STD-210B value at the primary level of interest, but is more realistic and less severe at all other levels.

APPENDIX

Tabulated Student's t-scores

TABLE 18. Student's t-scores for non-forcing mandatory levels at indicated North American Arctic cold extreme stations as computed from the full data samples and 10% extreme data samples using equation (5).

Mandatory level prediction test	Student's t-scores			
	Hall Beach, T_J(700)	Resolute, T_J(500)	Thule, T_J(300)	Thule, T_J(200)
1000 mb	17.306	10.971	1.570	16.362
850	157.648	63.095	4.090	65.823
700		128.288	9.889	45.967
500	66.065		15.446	38.910
300	95.571	12.539		29.500
200	72.996	18.336	43.853	
150	77.184	53.330	55.734	92.120
100	55.369	71.068	71.881	106.874
70	30.620	74.901	73.299	113.350

TABLE 19. Student's t-scores for non-forcing mandatory levels at indicated North American Arctic warm extreme stations as computed from the full data samples and 10% extreme data samples using equation (5).

Mandatory level prediction test	Student's t-scores		
	Alert, $T_J(200)$	Alert, $T_J(150)$	Thule, $T_J(100)$
1000 mb	5.155	3.148	5.122
850	27.369	23.869	16.855
700	32.395	26.896	20.638
500	45.093	29.831	28.217
300	16.006	2.905	4.818
200		27.907	24.030
150	24.921		7.045
100	18.204	25.841	
70	17.255	21.183	22.443

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13. ABSTRACT

A group of stations in the North American Arctic region have been analyzed for statistical determination of temperatures at mandatory pressure levels. For each station the temperature at a key level, peculiar to that station, has forced in at the first step and retained at each subsequent step, in the development of stepwise regression equations which predict temperatures at the mandatory levels. In general, eight-step predictions, in terms of inter-level thicknesses, were found to give optimum specification of the desired temperatures. The best estimate of the regional atmosphere, which is conditionally dependent upon the existence of an extreme 1% probability of the forcing level temperature, is obtained with a high degree of confidence.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
EXTREME TEMPERATURE						
FORCING-LEVEL TEMPERATURE						
MANDATORY PRESSURE LEVELS						
REGIONAL EXTREMES						
STEPWISE REGRESSION TECHNIQUE						
TEMPERATURE PREDICTORS						
THICKNESS CORRELATIONS						

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by regression methods.